Name: _____ Period ____

Unit 3A:

Linear Functions

Unit 3: Linear and Exponential Functions

Resources and Information at:

www.tinyurl.com/3layson

LEARNING-FOCUSED
Solutions That Work!

Student Learning Map

Unit 3: Linear & Exponential Functions

Part A: Linear Functions

Unit 3A: Linear Functions

Unit Essential Question:

Concept:	Concept:	Concept:
Lesson Essential Questions:	**Lesson Essential Questions:**	**Lesson Essential Questions:**
Vocabulary:	**Vocabulary:**	**Vocabulary:**

Additional Information/Resources:

www.usatestprep.com **school: newmanchesterga**
code: newton58

Lesson 3.1 – What is a Function?

Suppose…

Saturday you got a new video game for your X-Box 360. You played your new game for 3 hours on Saturday. You notice every time you press A, your player jumps.

Saturday

A ⟶ _____

On Sunday, you play your game again. You press A. What do you expect your player to do?

Your player does not jump, instead the player runs. Everything else works on the game so you continue to play.

Sunday

A ⟶ _____

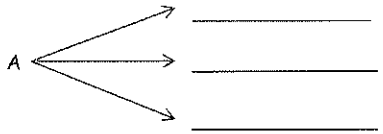
On Monday you play your game again. When you press A what do you expect your player to do?

Your player ducks. Are you willing to continue to play the game? Why or Why not?

Monday

A ⟶ _____

Mapping



_____

_____

_____

Relation vs. Function

_____: a relationship between two sets of data.

Ex:

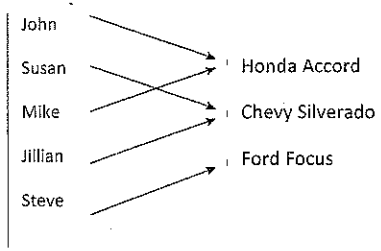
1. _____ on your block and the _____ they drive
2. { (1, 2), (2, 3), (4, 5), (1,1)}
3.

X	Y
0	
1	
2	

_____: a relationship between two sets of data where each _____ has only _____ _____.

Ex:

1. Neighbors on your block and the cars they drive (neighbor's _____ is assigned to a _____ and _____ of a car)

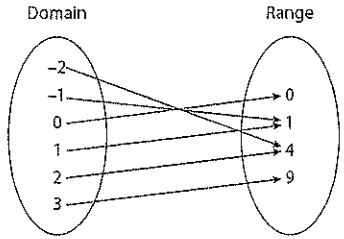


2. {(1,2), (2,3), (4,5), (6, 8)}

3.

X	Y
0	
1	
2	

4.



_____ – set of x-values that are valid for the function. AKA: _____

_____ – set of y-values that are valid for the function.

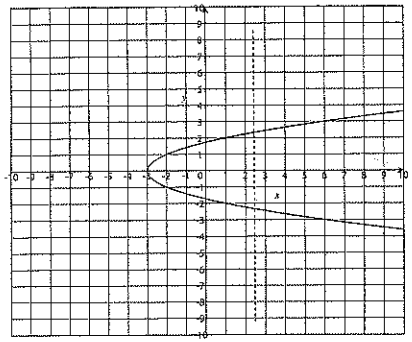
Graphs of Functions

_____ _____ _____: an imaginary _____ (up and down) line swept across the _____ to see if the line ever crosses _____ than _____ point on the graph at the same time.

- Function – crosses _____ _____ _____on the graph at the same time
- NOT Function – crosses _____ _____ _____ _____on the graph at the same time.

Function or Not?

Use the vertical line test to determine if each relation is a function.



Is the graph a function? _____

Why or Why not?

_____

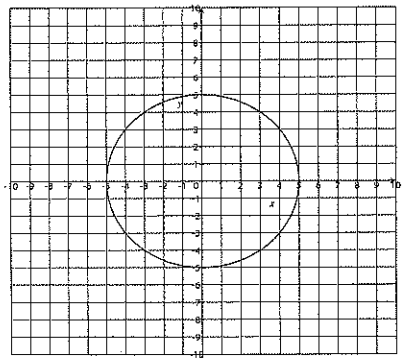
_____

_____

_____

_____

_____



Is the graph a function? _____

Why or Why not?

_____

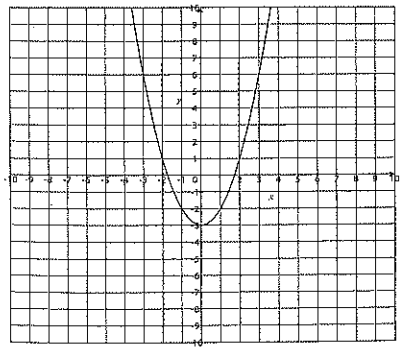
_____

_____

_____

_____

_____



Is the graph a function? _____

Why or Why not?

_____

_____

_____

_____

_____

_____

You Try!

1. {(1, 3), (2, 6), (1, 5), (3, 8)}

Is the relation a function? _____

Why or Why not?

_____

_____

_____

2. {(2, 4), (4, 8), (6, 12), (8, 16)}

Is the relation a function? _____

Why or Why not?

_____

_____

_____

3.

X	Y
-3	1
1	1
3	1

Is the table a function? _____

Why or Why not?

_____

_____

_____

4.

X	Y
2	1
2	5
3	1

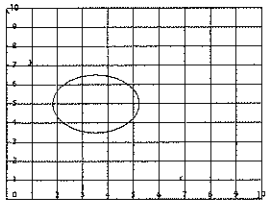
Is the table a function? _____

Why or Why not?

_____

_____

_____

5. 

Is the graph a function? _____

Why or Why not?

_____

_____

_____

Lesson 3.1 HW – What is a Function?

#1-2 Determine if each table represents a function? Explain why or why not?

1)

x	y
10	1
20	2
30	3
10	4
20	5
30	6

Explanation

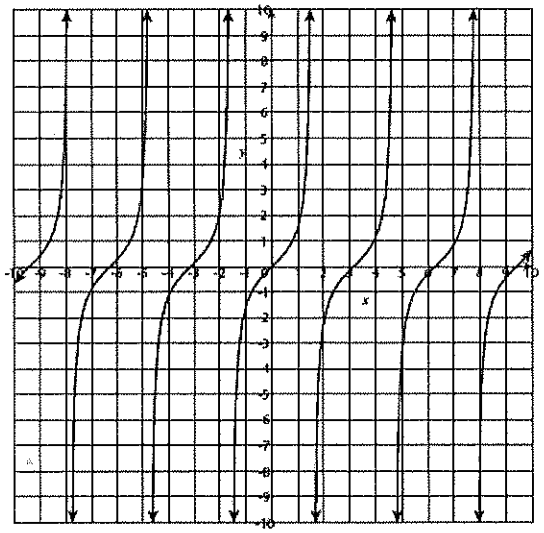
2)

x	y
1	0
2	1
4	2
8	3
16	4
32	5

Explanation

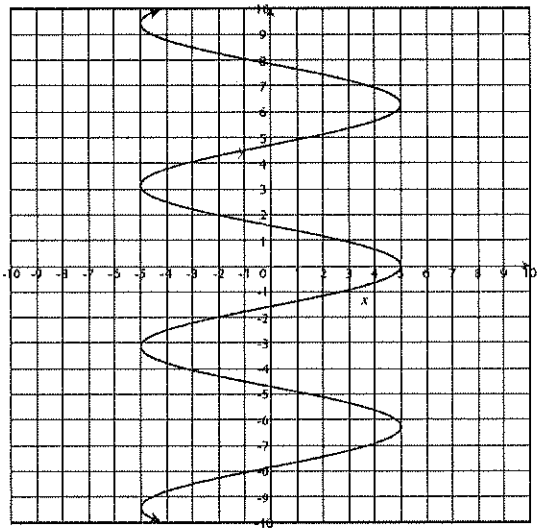
#3-4 Determine if each graph represents a function? Explain why or why not?

3)



Explanation

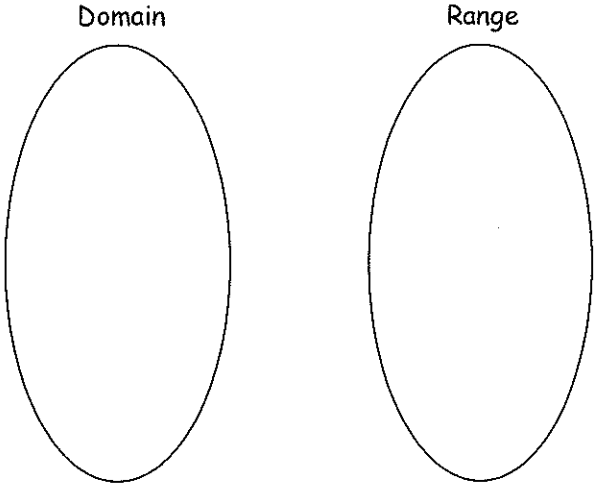
4)



Explanation

5) Complete the mapping diagram on the right for the relation. If it's a function, label it.

$\{(2,4),(2,6),(2,8),(2,10),(2,12),(2,14)\}$



6) Give a real life example of a function (besides a video game that's working correctly).

Example: _____

Input: _____

Output: _____

Lesson 3.2 Function Notation

- Recall that in a _____, every element of the domain is paired with exactly one element of the range. That is, for _____ value of x, there is exactly _____ value of y.
- _____ _____ is a way to name a function using $f(x)$ instead of y.
- Functions can be named using _____ _____, though f and g are used often.
- Functions can be evaluated at _____ _____ _____.
- To evaluate a function, _____ the values for the domain for all occurrences of x.
- $(x, f(x))$, is an _____ _____ of a function and a point on the graph of the function.

Steps for Evaluating Functions

Step 1: _____

_____

Step 2: _____

_____

Example 1: Evaluate $f(x) = 4x - 7$ over the domain $\{1, 2, 3, 4\}$. What is the range?

$f(1) =$ $f(2) =$

$f(3) =$ $f(4) =$

The range is: {____, ____, ____, ____}

You Try!

1. $f(x) = \frac{x}{2} - 6$ for the domain $\{2, 4, 6, 8\}$

The range is: {____, ____, ____, ____}

Example 2: Complete the input-output table for the function $f(x) = -3x + 5$.

Step 1: First we _____ $f(-1)$ and $f(3)$.

$f(-1) =$ $f(3) =$

Input (x)	Output (f(x))
-1	
	5
3	
	-10

Step 2: _____ these values into our table.

Step 3: We need to find when $f(x) = 5$ and when $f(x) = -10$. To do this, we need to _____ the equation $-3x + 5$ _____ to our given _____.

$5 = -3x + 5$ $-10 = -3x + 5$

Step 4: Plug these _____ into our _____.

You Try!

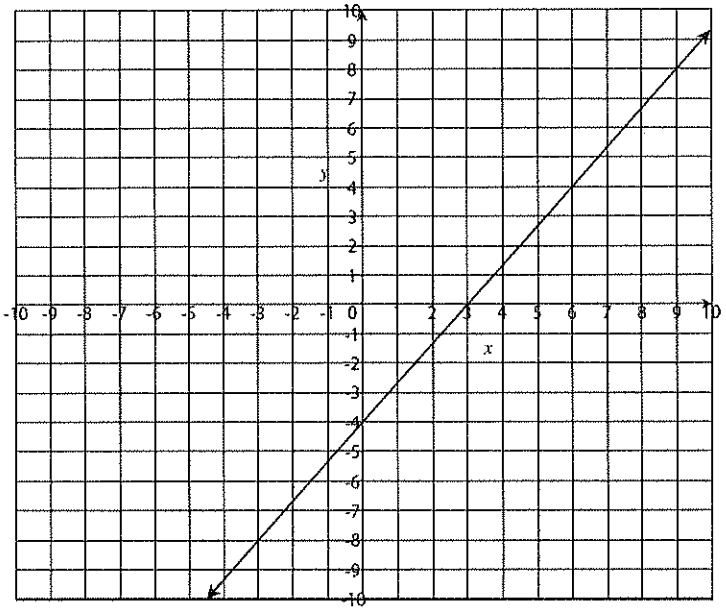
2. Complete the input-output table for the function $f(x) = -x + 3$

Input (x)	Output (f(x))
0	
	1
5	
	6

Example 3:

a. Look at the graph of $f(x)$, what is $f(-3)$?

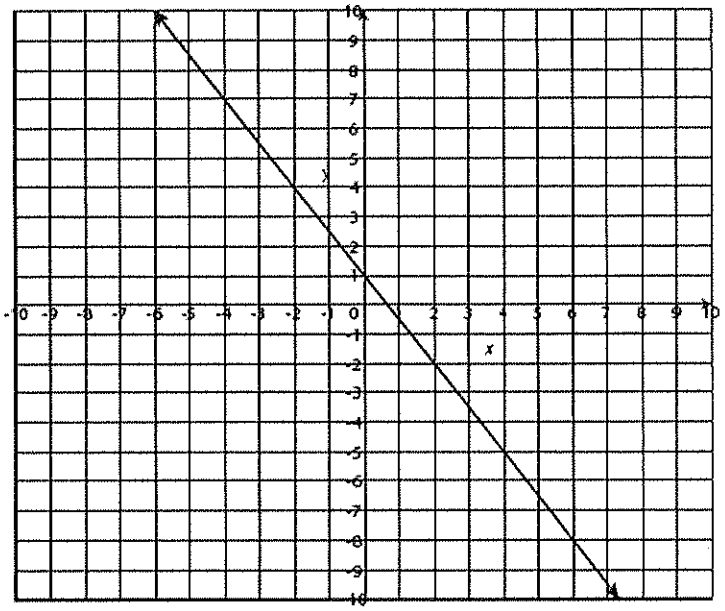
b. For what value of x will $f(x) = 4$?



You Try! 3

a. Look at the graph of $f(x)$, what is $f(6)$?

b. For what value of x will $f(x) = -5$?



Example 4: Cindy has a steady babysitting job. The total charge for a babysitting job can be represented by the function $c(t) = 9t + 2$, where t is the number of hours. **Evaluate $c(5)$ and interpret the results.**

$c(5) =$ _____

Interpretation: _____

You Try!

4. The local Italian restaurant has had a steady increase in customers and is, therefore, hiring people $h(x) = 4x + 10$, where x is in months. **Evaluate $h(7)$ and interpret the results.**

$h(7) =$ _____

Interpretation: _____

Lesson 3.2 HW – Function Notation

1) Evaluate $f(x) = 2x - 7$ over the domain $\{2, 4, 6, 8\}$. What is the range of $f(x)$? **Show your work!**

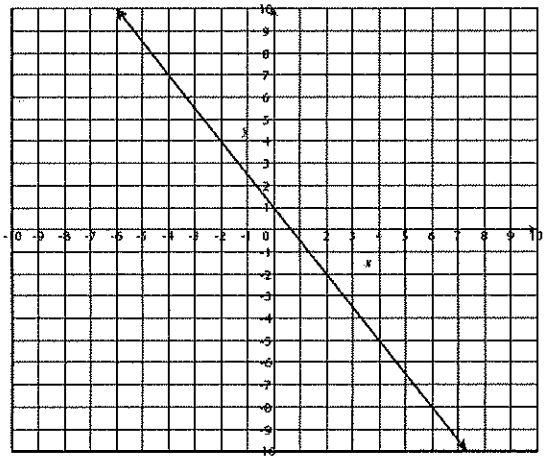
Range: {_____}

2) Complete the input-output table for the given function. **Show your work!**

$g(x) = 3x + 10$

Input x	Output $g(x)$
	10
6	
12	
	82

3) Use the graph of $f(x)$ below.



$f(-4) =$ _____

If $f(x) = 8$, $x =$ _____

4) A bookstore has seen a steady decline in sales. As a result, the store is laying off employees at a steady rate. The function of the decline in employees is $h(x) = -2x + 56$, where x is in months. **Evaluate $h(3)$ and interpret the results.** **Show your work!**

$h(3) =$ _____

Interpretation: _____

10

Lesson 3.3 – Constant Rate of Change

Introduction

- A _____ _____ _____ is a ratio that describes how much one quantity changes with respect to the change in another quantity of the function.
- With linear functions the rate of change is called the _____. The slope of a line is the ratio of the change in *y*-values to the change in *x*-values. Formula: m = _____
- Linear functions have a *constant* rate of change, meaning values _____ or _____ at the same rate over a period of time.

Recall

Calculating *Constant* Rate of Change *(slope)* from a Table

1. Choose _____ points from the table (highest and lowest points).
2. Assign one point to be (___, ___) and the other point to be (___, ___).
3. _____ the values into the slope formula:
$$m = \frac{y_2 - y_1}{x_2 - y_2}$$
4. The result is the rate of change for the interval between the two points chosen. *The rate of change between any two points of a linear function will be equal.
5. _____ your answer for the context of the problem.

Example 1

To raise money, students plan to hold a car wash. They ask some adults how much they would pay for a car wash. The table on the right shows the results of their research. What is the rate of change for their results?

Carwash Price (*x*)	Number of Customers (*f*(*x*))
$4	120
$6	106
$8	92
$10	78

1. Choose two points.

2. Assign the points $(x_1, y_1), (x_2, y_2)$

3. Substitute the values into the slope formula.

4. Rate of Change:

5. Interpretation:

Recall

Estimating Constant Rate of Change *(slope)* from a Graph

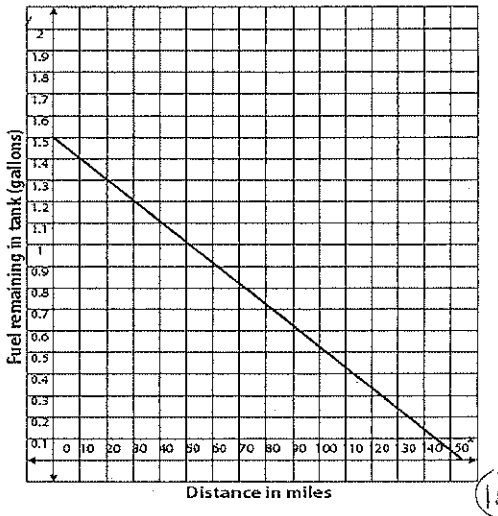
1. Pick two points from the _____.
2. Identify (x_1, y_1) as one point and (x_2, y_2) as the other point.
3. Substitute (x_1, y_1) and (x_2, y_2) into the _____ formula to calculate the rate of change. $m = \frac{y_2 - y_1}{x_2 - y_2}$
4. The result is the _____ rate of change *(slope)* for the graph. *The rate of change between any two points of a linear function will be equal.
5. _____ your answer for the context of the problem.

Example 2

The graph to the right compares the distance a small motor scooter can travel in miles to the amount of fuel used in gallons. What is the rate of change for this scenario?

1. Choose two points.

2. Assign the points $(x_1, y_1), (x_2, y_2)$



3. Substitute the values into the slope formula.

4. Rate of Change:

5. Interpretation:

You Try 1

Calculate the constant rate of change(slope) from the table of values.

You are a cashier at McDonald's and you are paid per hour. After two hours of work, you earn $15. Then after 6 hours of work, you have earned $45. How do you find your hourly rate?

Hours worked (x)	Money earned (f(x))
2	15
4	30
6	45
8	60

1. Choose two points.

2. Assign the points $(x_1, y_1), (x_2, y_2)$

3. Substitute the values into the slope formula.

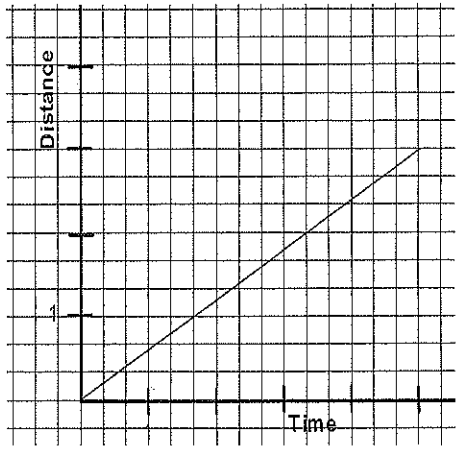
4. Rate of Change:

5. Interpretation:

You Try 2

Calculate the constant rate of change (slope) for the graph.

Tim is walking to his grandmother's house and is using a smartphone app to calculate his speed and the distance he ran. The app gives him the graph to the right. What is his speed (rate of change)?



1. Choose two points.

2. Assign the points $(x_1, y_1), (x_2, y_2)$

3. Substitute the values into the slope formula.

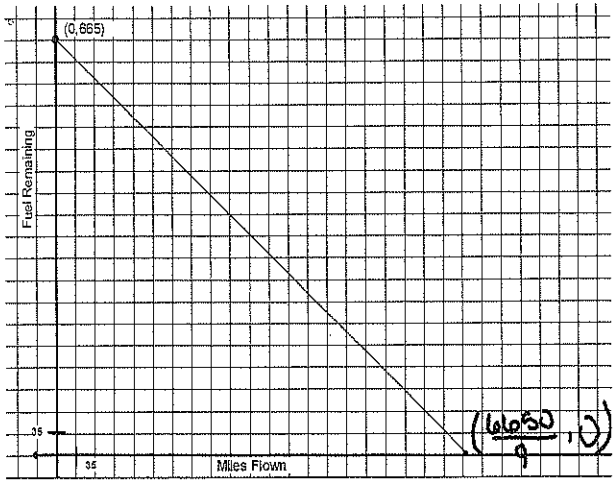
4. Rate of Change:

5. Interpretation:

Lesson 3.3 HW- Constant rate of change

Directions: Find the rate of change and interpret your answer for each problem. You must show all work to receive credit!!!

1. The Beechcraft 1900D us a commuter airplane with a fuel capacity of 665 gallons. The graph given below represents the function where x represents miles flown and f(x) represents the e amount of fuel remaining. What is the rate of change for this scenario and interpret your answer?



Rate of Change: _____

Interpretation: _____

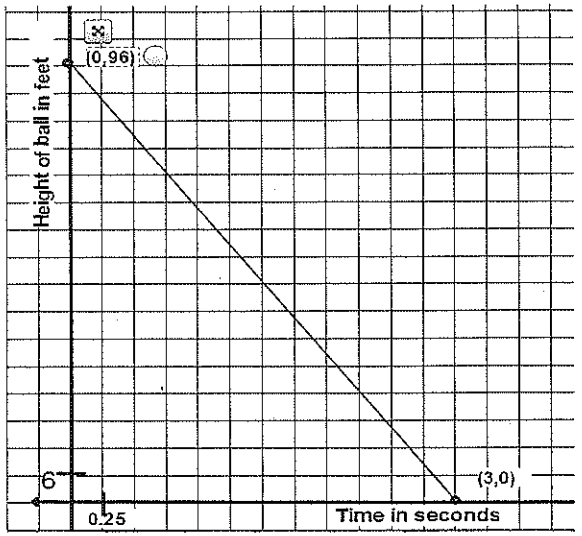
2. The table below lists common Celsius to Fahrenheit degree conversions. What is the rate of change for this function?

°C (x)	°F (f(x))
0	32
10	50
20	68
30	86
40	104

Rate of Change: _____

Interpretation: _____

3. The velocity of a ball thrown directly upward can be modeled by the given graph below, where x represents time in seconds and f(x) represents the height of the ball in feet. What is the rate of change for this scenario and interpret your answer?



Rate of Change: _____

Interpretation: _____

4. The table below lists the iPhone 5S production at one of the Apple factories in California, where x represents hours and f(x) represents the amount of iPhones produced after so many hours. What is the amount of iPhones produced per hour (rate of change)?

Hours (x)	iPhones Made (f(x))
3	210
6	420
9	630
12	840
15	1050

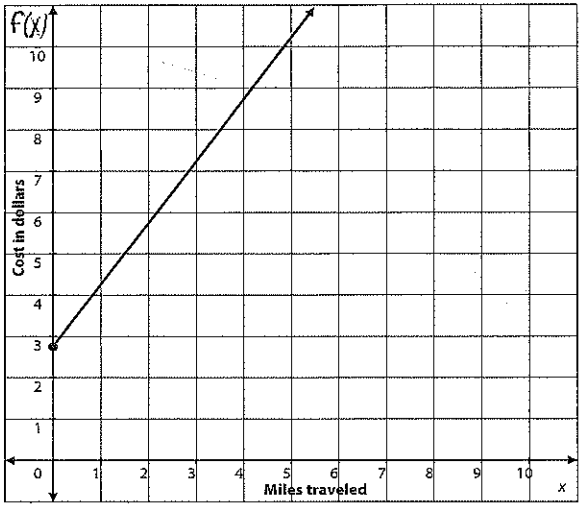
Rate of Change: _____

Interpretation: _____

Lesson 3.4 – Characteristics of Linear Functions

Example 1

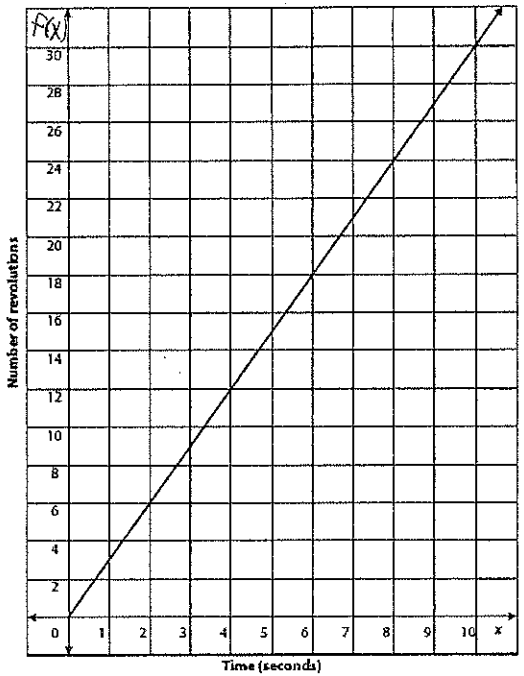
A taxi company in Atlanta charges $2.75 per ride plus $1.50 for every mile driven. Determine the key features of this function.



1. Type of function	
2. Domain and Range	Domain: Range:
3. Y-intercept	
4. Intervals of Increase/Decrease	
5. Extrema (Max /Min)	Maximum: Minimum:
6. Rate of Change	

Example 2

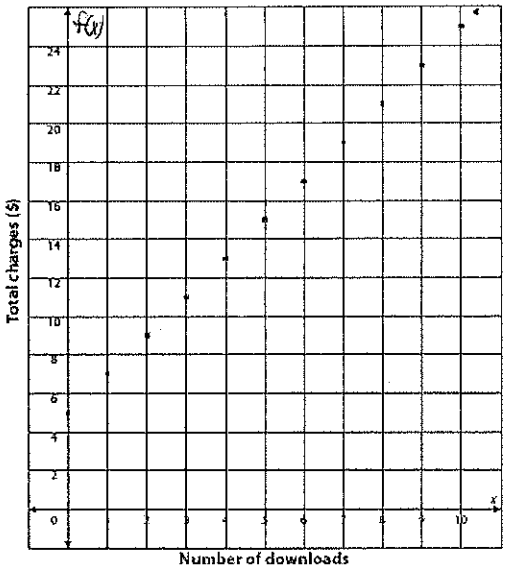
A gear on a machine turns at a rate of 3 revolutions per second. Identify the key features of the graph of this function.



1. Type of function	
2. Domain and Range	Domain: Range:
3. Y-intercept	
4. Intervals of Increase/Decrease	
5. Extrema (Max /Min)	Maximum: Minimum:
6. Rate of Change	

Example 3

An online company charges $5 a month plus $2 for each movie you decide to download. Identify the key features of the graph of this function.



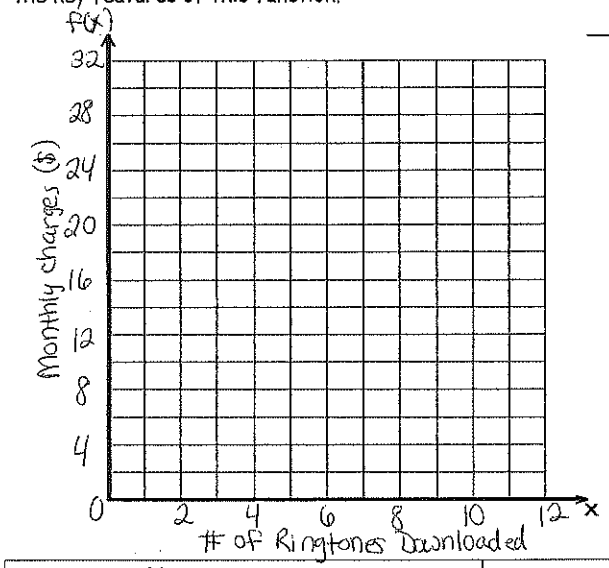
1. Type of function	
2. Domain and Range	Domain: Range:
3. Y-intercept	
4. Intervals of Increase/Decrease	
5. Extrema (Max /Min)	Maximum: Minimum:
6. Rate of Change	

Example 4

A ringtone company charges $15 a month plus $2 for each ringtone downloaded. Write an equation to represent the monthly charges of the ringtone company. Then graph and identify the key features of this function.

_____

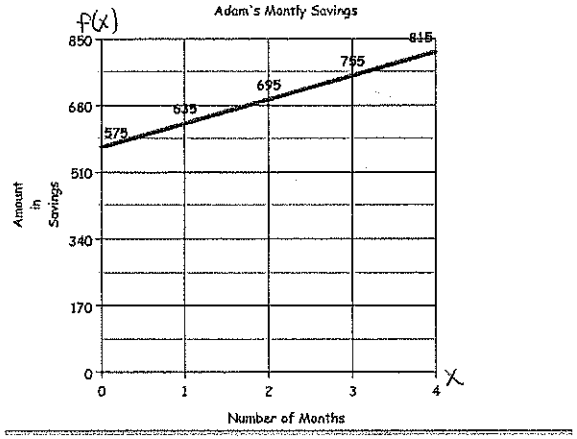
Equation



1. Type of function	
2. Domain and Range	Domain: Range:
3. Y-intercept	
4. Intervals of Increase/Decrease	
5. Extrema (Max /Min)	Maximum: Minimum:
6. Rate of Change	

You Try 1

The starting balance of Adam's savings account is $575. Each month, Adam deposits $60.00. Adam wants to keep track of his deposits so he creates the graph below from the following equation: f(x) = 60x + 575, where x = number of months.

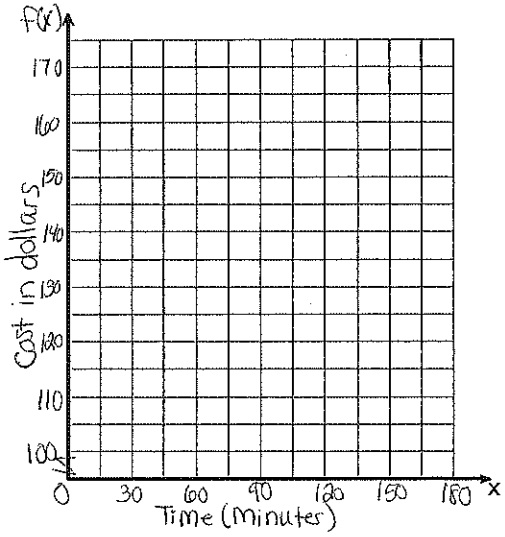


Identify the following information:

1. Type of function	
2. Domain and Range	Domain: Range:
3. Y-intercept	
4. Intervals of Increase/Decrease	
5. Extrema (Max /Min)	Maximum: Minimum:
6. Rate of Change	

You Try 2

The cost of an air conditioner is $110. The cost to run the air conditioner is $0.35 per minute. The table below represents this relationship. Graph and identify the key features of this function.

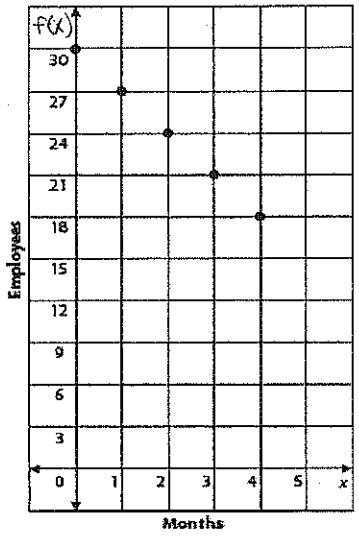


Minutes (x)	Cost in dollars (f(x))
0	110.00
30	120.50
60	131.00
90	141.50
120	152.00

1. Type of function	
2. Domain and Range	Domain: Range:
3. Y-intercept	
4. Intervals of Increase/Decrease	
5. Extrema (Max /Min)	Maximum: Minimum:
6. Rate of Change	

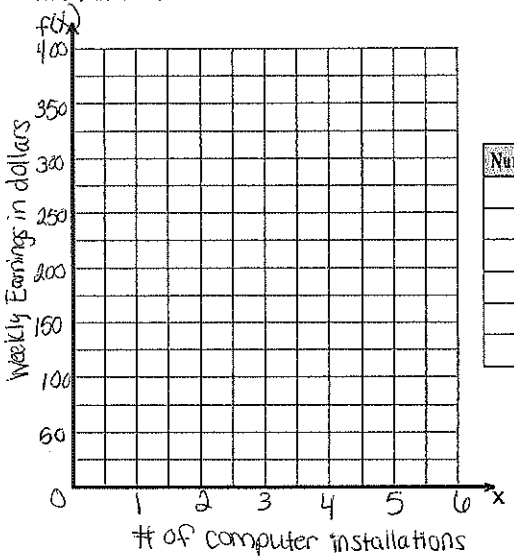
Lesson 3.4 HW – Characteristics of Linear Functions

1. A small bookstore is downsizing and has lost employees at a steady rate. The company started with 30 employees, but has lost 3 employees each month for the past 4 months. Identify the key features of the graph of this function.



1. Type of function	
2. Domain and Range	Domain: Range:
3. Y-intercept	
4. Intervals of Increase/Decrease	
5. Extrema (Max /Min)	Maximum: Minimum:
6. Rate of Change	

2. Alice receives a base weekly salary of $150 plus a commission of $45 for each computer she installs. The table below represents this relationship. Graph and identify the key features of this function.



Number of computer installations (x)	Weekly earnings in dollars (f(x))
0	150
1	195
2	240
3	285
4	330
5	375

1. Type of function	
2. Domain and Range	Domain: Range:
3. Y-intercept	
4. Intervals of Increase/Decrease	
5. Extrema (Max /Min)	Maximum: Minimum:
6. Rate of Change	

Lesson 3.5 Identifying Solutions

Let's Review

- A _____ to a system of equations is a value that makes both equations true.
- The _____ where two lines intersect is a solution to _____ equations.
- Remember that in real-world problems, the _____ of the equation is the amount that describes the rate of change, and the _____ is the amount that represents the initial value.

Introduction

Words to know for business problems:

- _____ - the money spent to purchase your product or equipment.
- _____ - the total money obtained from selling your product.
- _____ - the expenses subtracted from the income.
- _____ - _____ _____ - the point where the expenses and the income are equal. In other words you don't make money nor lose money…your profit is $0.

You can find the x-coordinate of the intersection of two linear functions in 3 different ways:

1. By _____ their graphs
2. Making a _____
3. Setting the functions _____ to each other (algebraically)

Example 1:

Aly and Dwayne work at a water park and have to drain the water from the small pool at the bottom of their ride at the end of the month. Each uses a pump to remove the water.
Aly's pool has 35,000 gallons of water in it and drains at a rate of 1,750 gallons a minute.
Dwayne's pool has 30,000 gallons of water in it and drains at a rate of 1,000 gallons a minute.
After approximately how many minutes will Aly and Dwayne's pools have the same amount of water in them?

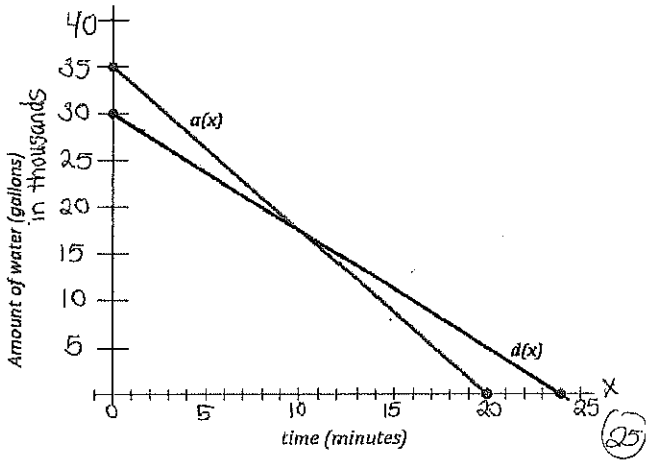
a(x) = _____

d(x) = _____

Estimate of Point of Intersection: (___ , ___)

x-coordinate: _____

After _____ minutes Aly and Dwayne will have the same amount of water in their pools.



Example 2:

A large cheese pizza at Paradise Pizzeria costs $6.80 plus $0.90 for each topping.
The cost of a large cheese pizza at Geno's Pizza is $7.30 plus $0.65 for each topping.
How many toppings need to be added to a large cheese pizza from Paradise and Geno's in order for the pizzas to cost the same, not including tax?

# of toppings	Paradise Pizzeria	Geno's Pizza
x	_____	_____
0		
1		
2		

After _____ toppings the pizzas will cost the same at $_____.

Example 3:

Eric sells model cars from a booth at a local flea market. He purchases each model car from a distributor for $12, and the flea market charges him a booth fee of $50. Eric sells each model car for $20. How many model cars must Eric sell in order to reach the break-even point?

Eric's expenses: _____ Eric's income: _____

e(x) = f(x)

_____ = _____

_____ = _____

_____ = x

Eric must sell more than _____ model cars in order to break-even.

Eric's profit: _____

_____ = _____

_____ = _____

_____ = _____

You Try 1: *Solve using a graph*

Chen starts his own lawn mowing business. He initially spends $180 on a new lawnmower. For each yard he mows, he receives $20 and spends $4 on gas. If x represents the # of lawns, then let Chen's expenses be modeled by the function e(x) = 4x + 180 and his income be modeled by the function m(x) = 20x. How many lawns must Chen mow to break-even?

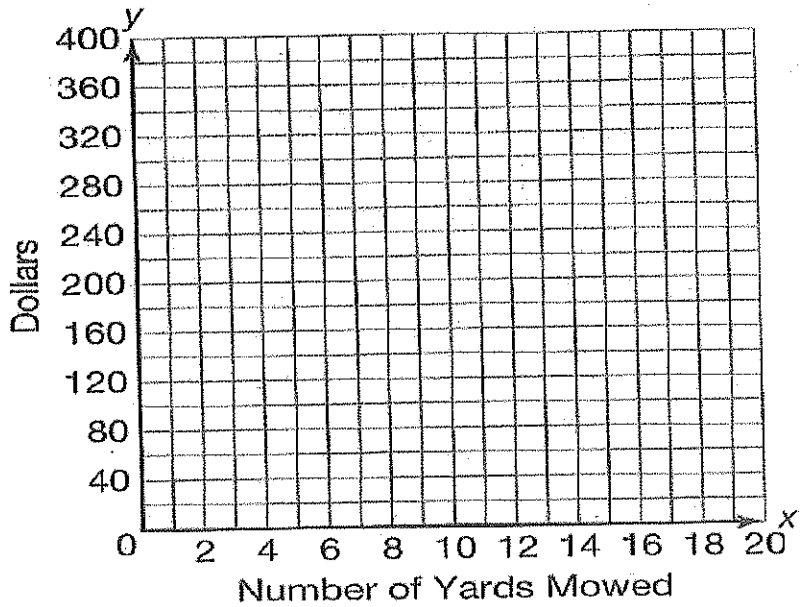
e(x) = _____

m(x) = _____

Estimate of Point of Intersection: (___ , ___)

x-coordinate: _____

After _____ lawns are mowed, Chen will break-even.



You Try 2: *Solve using a table*

Olivia is building birdhouses to raise money for a trip to Hawaii. She spends a total of $30 on the tools needed to build the houses. The material to build each birdhouse costs $3.25. Olivia sells each birdhouse for $10. If x represents the # of birdhouses, then let Olivia's expenses be modeled by the function e(x) = 3.25x + 30 and her income be modeled by the function b(x) = 10x. How many birdhouses must Olivia sell to break-even?

# of birdhouses x	Expenses _____	Income _____

Olivia must sell _____ birdhouses in order to break even.

You Try 3: *Solve algebraically*

Text Away cell phone company charges a flat rate of $30 per month plus $0.20 per text.
It's Your Dime cell phone company charges a flat rate of $20 per month plus $0.40 per text.
If x represents the # of texts, then let your Text Away bill be modeled by the function t(x) = .20x + 30 and Your Dime bill be modeled by the function d(x) = .40x + 20
How many texts must you send before your bill for each company will be the same?

Text Away: _____ It's Your Dime: _____

t(x) = d(x)

_____ = _____

_____ = _____

_____ = x

You must send _____ text messages in order for your bill for each company to be the same.

Lesson 3.5 HW - Identifying Solutions

Directions: Identify and interpret the solution where the functions are equal. Solve using the indicated method and show all work were necessary.

1. Jeff and Sherri are packing boxes for Operation Christmas Child. Jeff has packed 24 boxes and is packing 10 boxes per hour. Sherri has packed 31 boxes and is packing 8 boxes per hour. If x represents boxes, then Jeff's function can be modeled as j(x) = 10x +24 while Sherri's function can be modeled as s(x) = 8x +31. In how many hours will they have packed the same number of boxes?

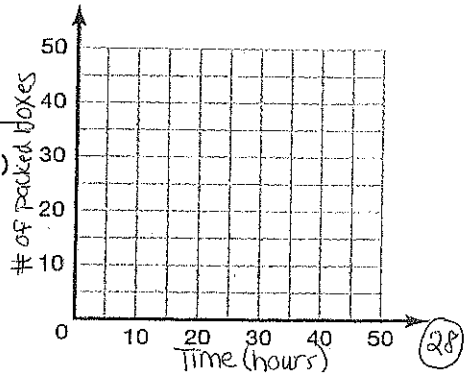
Graph the functions.

j(x) = _____ s(x) = _____

Estimate of Point of Intersection: (___ , ___)

x-coordinate: _____

After _____ hours, Jeff and Sherri will have packed the same number of boxes.



Lesson 3.5 HW - Identifying Solutions *(continued)*

2. Ling decided to sell cupcakes at the county fair. Her ingredients cost her about 25 cents per cupcake. Renting a booth costs $30 per day. She sells each cupcake for $1. Ling's expenses can be modeled by the function c(x) = 0.25x + 30.00. Her income can be modeled by the function f(x) = 1.00x.

a. How many cupcakes must she sell before she turns a profit?

Expenses: _____

Income: _____

c(x) = f(x)

_____ = _____

_____ = _____

_____ = x

Ling must sell _____ cupcakes before she turns a profit.

b. Using the above functions, write a function to represent Ling's profit, P(x) = c(x) – f(x)

P(x) = _____

3. The Spanish Club is selling boxes of fruit as a fundraiser. The fruit company charges the Spanish club $7.50 for each box of fruit and a shipping and handling fee of $100 for the entire order. The Spanish Club sells each box of fruit for $15. If x represents boxes of fruit, then the club's expenses can be modeled by the function f(x) = 7.50x + 100 and their income function g(x) = 15x.

a. How many boxes of fruit must the Spanish Club sell in order to break-even?

# of boxes of fruit	Expenses	Income
x	_____	_____

The Spanish Club must sell _____ boxes of fruit in order to break even.

b. Using the above functions, write a function to represent the Spanish Club's profit, P(x) = g(x) – f(x)

P(x) = _____