Name: ____________________ Period ____

Unit 3B: Exponential Functions

Unit 3: Linear and Exponential Functions

Resources and Information at:

www.tinyurl.com/9layson

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Student Learning Map

Unit 3: Linear & Exponential Functions

Part B: Exponential Functions

Unit 3B: Exponential Functions

Unit Essential Question: How do we use linear and exponential functions to interpret, analyze, and model situations?

Concept:	Concept:	Concept:

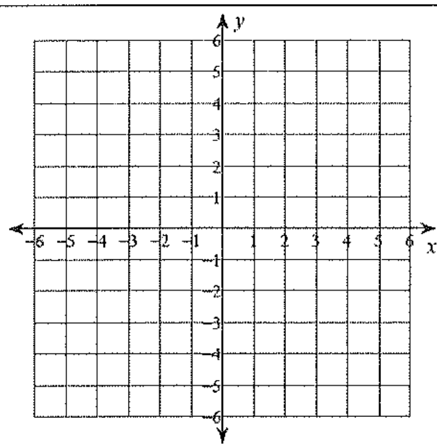
Lesson Essential Questions:	Lesson Essential Questions:	Lesson Essential Questions:

Vocabulary:	Vocabulary:	Vocabulary:

Additional Information/Resources:

www.usatestprep.com **school: newmanchesterga**
code: newton58

Lesson 3.6 - Graphing & Identifying Key Features of Exponential Functions

Definitions and Givens	General form $f(x) = ab^x + k$ a = initial value that determines the shape a > 1 stretch; 0 < a < 1 shrink; – a = reflection b = growth if the value is > 1 b = decay if the value is between 0 and 1 k = horizontal asymptote & vertical shift	Example 1: $f(x) = 2^x$ a = _____ Reflection? ______ b = _____ Growth or Decay? _________
Asymptote	Line that a graph approaches but __________ __________.	k = _____ Horizontal asymptote is the line $y =$ _____
y – intercept	The ________ where the graph crosses the ____ axis. The value of x is ___ at this point.	Substitute ___ for x and solve to find the ________________. Y –intercept = ______
Graph		x | f(x) -2 | -1 | 0 | 1 | 2 |
Domain	The collection of all x-values (inputs). For exponential functions, the domain will always be all real numbers _____.	________________ = all real numbers because any number can be used as x.
Range	The collection of all y-values (outputs). + a : Range is all numbers ____ asymptote – a : Range is all numbers ____ asymptote	__________= all numbers > asymptote $y >$ __
End behavior	What happens at the ends of the graph. Exponential functions have ____ end behaviors. One towards + or – infinity and one towards the horizontal asymptote.	*Left:* As $x \to -\infty, y \to$ ______ *Right:* As $x \to +\infty, y \to$ ______

2. $f(x) = \left(\frac{1}{2}\right)^x$

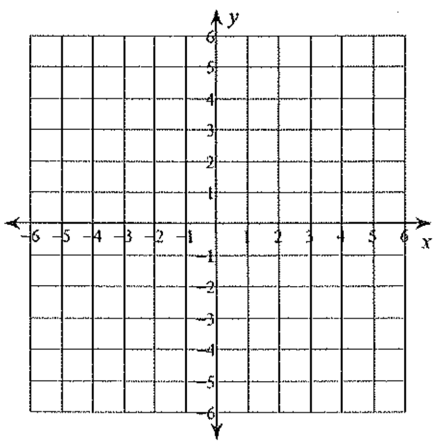
a = _____ Reflection? _____________

b = _____ Growth or Decay? _____________

k = _____

Horizontal asymptote is y = ___

y –intercept = ______



x	f(x)
-2	
-1	
0	
1	
2	

Domain = ___________________

Range = all numbers ______ < or > asymptote

y ____ ______
< or >

Left: As $x \to -\infty, y \to$ ______

Right: As $x \to +\infty, y \to$ ______

3. $f(x) = 3^x + 1$

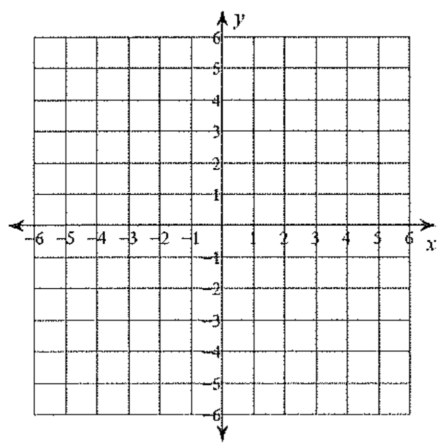
a = _____ Reflection? _____________

b = _____ Growth or Decay? _____________

k = _____

Horizontal asymptote is y = ___

y –intercept = ______



x	f(x)
-2	
-1	
0	
1	
2	

Domain = ___________________

Range = all numbers ______ < or > asymptote

y ____ ______
< or >

Left: As $x \to -\infty, y \to$ ______

Right: As $x \to +\infty, y \to$ ______

4. $f(x) = -1(2)^x + 3$

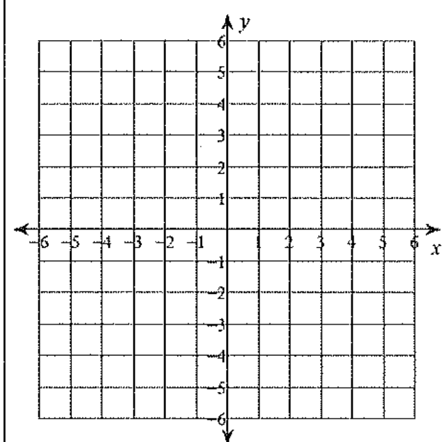
a = _____ Reflection? _____________

b = _____ Growth or Decay? _____________

k = _____

Horizontal asymptote is y = ___

y –intercept = ______



x	f(x)
-2	
-1	
0	
1	
2	

Domain = ___________________

Range = all numbers ______ < or > asymptote

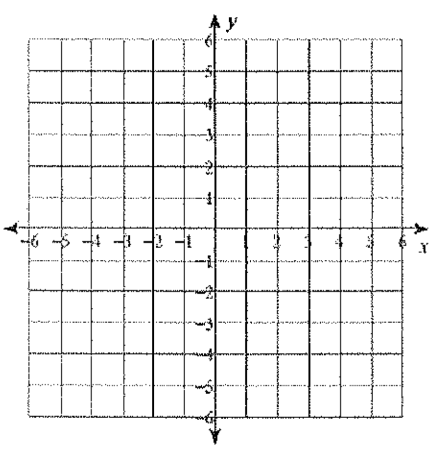
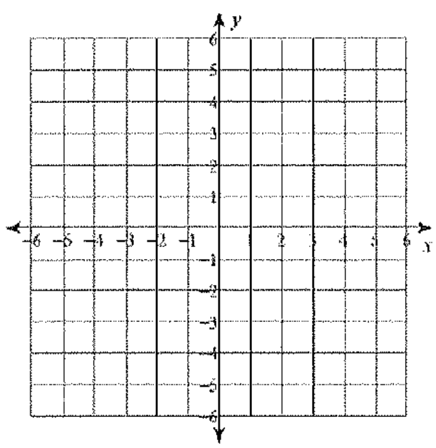
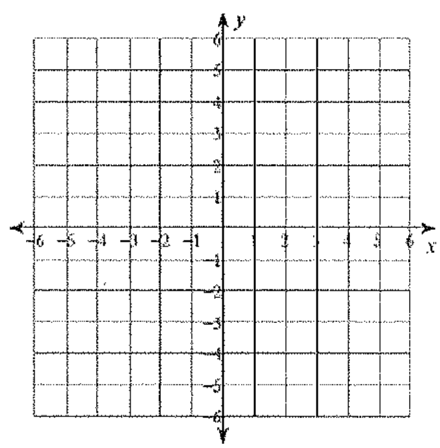
y ____ ______
< or >

Left: As $x \to -\infty, y \to$ ______

Right: As $x \to +\infty, y \to$ ______

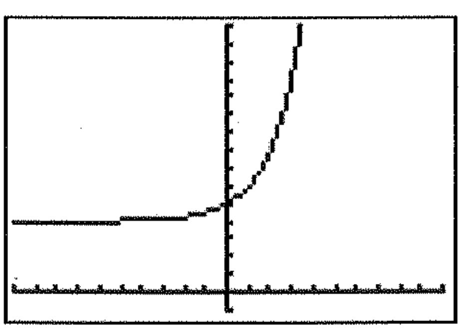
Lesson 3.6a HW - Key Features of Exponential Functions

#1-3 Directions: Complete the table for each function below and sketch its graph.

1. $f(x) = \left(\frac{1}{3}\right)^x$	2. $f(x) = -1(4)^x + 1$	3. $f(x) = 2^x - 1$
a = _____ Reflection? ____________	a = _____ Reflection? ____________	a = _____ Reflection? ____________
b = _____ Growth or Decay? ____________	b = _____ Growth or Decay? ____________	b = _____ Growth or Decay? ____________
k = _____ Horizontal asymptote is y = ___	k = _____ Horizontal asymptote is y = ___	k = _____ Horizontal asymptote is y = ___
y –intercept = ______	y –intercept = ______	y –intercept = ______
		

#4-5 Directions: Choose the correct equation that matches the given graph.

4.



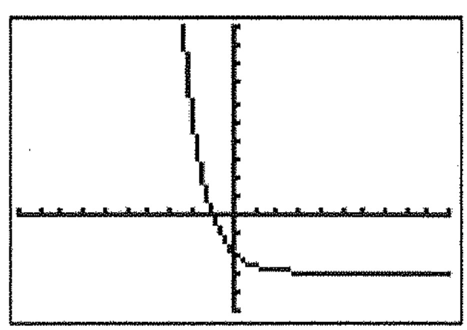
A. $f(x) = 2^x + 4$

B. $f(x) = \left(\frac{1}{2}\right)^x + 4$

C. $f(x) = 2^x + 5$

D. $f(x) = \left(\frac{1}{2}\right)^x + 5$

5.



A. $f(x) = 3(3)^x$

B. $f(x) = 3\left(\frac{1}{3}\right)^x$

C. $f(x) = 3^x - 3$

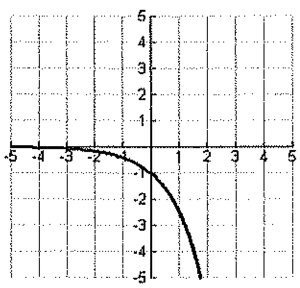
D. $f(x) = \left(\frac{1}{3}\right)^x - 3$

Name: ______________________ Date: ____________

Lesson 3.6b HW - Key Features of Exponential Functions

① $f(x) = -2^x$ Reflected?

Growth or Decay? | y-intercept =



Asymptote: ______

End behavior:

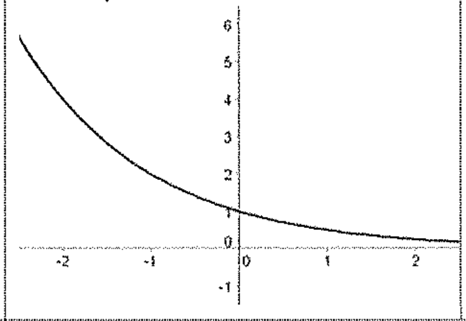
As $x \to -\infty, y \to$ ____

As $x \to \infty, y \to$ ____

Find the Domain ______________ and Range __________

② $f(x) = (½)^x$ Reflected?

Growth or Decay? | y-intercept =



Asymptote: ______

End behavior:

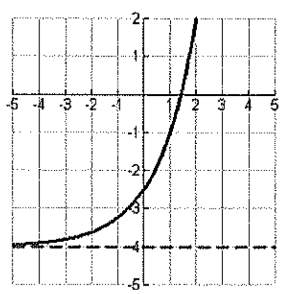
As $x \to -\infty, y \to$ ____

As $x \to \infty, y \to$ ____

Find the Domain ______________ and Range __________

③ $f(x) = 1.5(2)^x - 4$ Reflected?

Growth or Decay? | y-intercept =



Asymptote: ______

End behavior:

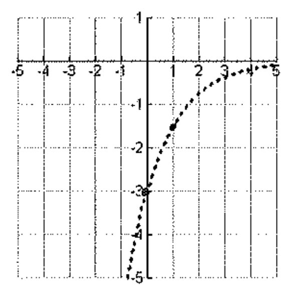
As $x \to -\infty, y \to$ ____

As $x \to \infty, y \to$ ____

Find the Domain ______________ and Range __________

④ $f(x) = -3(½)^x$ Reflected?

Growth or Decay? | y-intercept =



Asymptote: ______

End behavior:

As $x \to -\infty, y \to$ ____

As $x \to \infty, y \to$ ____

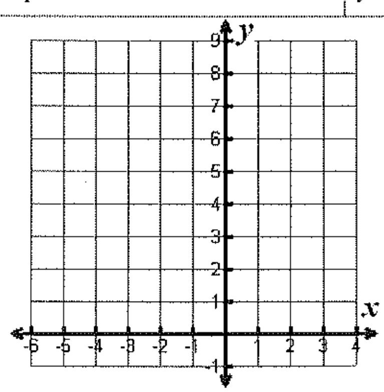
Find the Domain ______________ and Range __________

Complete the tables and graph the functions below. Then identify the key features.

⑤ $f(x) = 2(2)^x - 1$

Reflected? | Growth or Decay?

Asymptote: | y-intercept =



x	$f(x)$
-2	
-1	
0	
1	
2	

Domain ________ Range ________

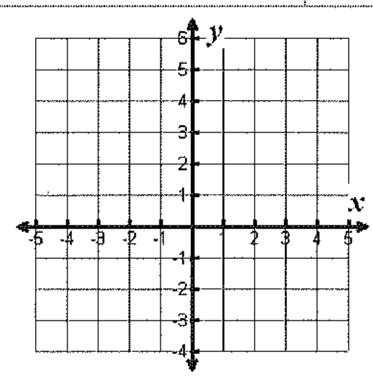
End Behavior: As $x \to -\infty, y \to$ ____

As $x \to +\infty, y \to$ ____

⑥ $f(x) = -\left(\frac{1}{2}\right)^x + 5$

Reflected? | Growth or Decay?

Asymptote: | y-intercept =



x	$f(x)$
-2	
-1	
0	
1	
2	

Domain ________ Range ________

End Behavior: As $x \to -\infty, y \to$ ____

As $x \to +\infty, y \to$ ____

Lesson 3.7 – Average Rate of Change

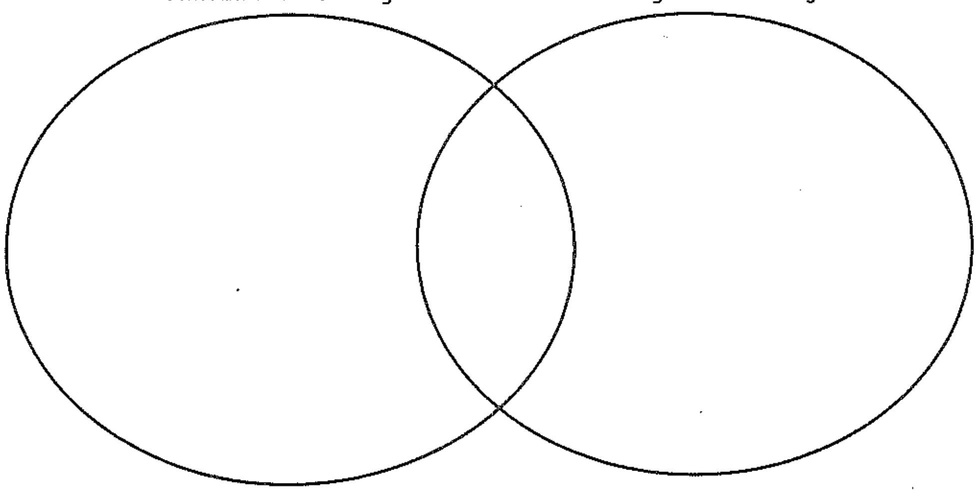
Introduction

In the previous unit, we found the ____________ rate of change of linear equations and functions using the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$

The slope of a line is the ratio of the change in y-values to the change in x-values.

The _______ _____ __________ can be determined from graphs, tables, and equations themselves. In this lesson, we will extend our understanding of the slope of linear functions to that of intervals of ____________ ____________.

Constant Rate of Change VS. Average Rate of Change



Key Concepts

Finding the rate of change of a non-linear function, in our case, exponential functions, is very similar to that of a linear function. You still use the slope formula to calculate the rate of change, but you are told by the *interval* which two points to use for (x_1, y_1) and (x_2, y_2)

- An ____________ is a continuous portion of a function.
- The rate of change of an interval is the _________ rate of change for that period.
- Intervals can be noted using the format $[a, b]$, where a represents the _________ ___ value of the interval and b represents the ________ ____ value of the interval. Another way to state the interval is $a \le x \le b$.

- For example, the interval [2, 7] means the portion of the function where x = 2 ___________ x = 7. You would use the points (2, y) & (7, y) in the _________ formula when calculating the rate of change for the function.

Steps to Calculating Average Rate of Change
1. Identify the ____________ to be observed.
2. Identify (x_1, y_1) as the ___________ point of the interval.
3. Identify (x_2, y_2) as the ___________ point of the interval.
4. Substitute (x_1, y_1) and (x_2, y_2) into the slope formula to calculate the rate of change. $m = \frac{y_2 - y_1}{x_2 - x_1}$
5. The result is the __________ rate of change for the interval between the two points identified.
6. ____________ your answer in the context of the problem.

Example 1:

In 2008, about 66 million U.S. households had both landline phones and cell phones. Use the table to calculate and interpret the rate of change for the interval [2008, 2011].

Year (x)	Households in millions ($f(x)$)
2008	66
2009	61
2010	56
2011	51

1. Determine the interval to be observed: [______ , _______]
2. Determine (x_1, y_1): (_____ , _____)
3. Determine (x_2, y_2): (_____ , _____)
4. Substitute points into the slope formula to calculate the average rate of change:

Average Rate of Change:__________

5. Interpretation:____

____

You Try 1

The table below represents a type of bacteria that doubles every 36 hours. A Petri dish starts out with 12 of these bacteria. Calculate and interpret the average rate of change over the interval [2,5].

Days (x)	Amount of bacteria ($f(x)$)
0	12
1	19
2	30
3	48
4	76
5	121
6	192

Rate of Change:____________.

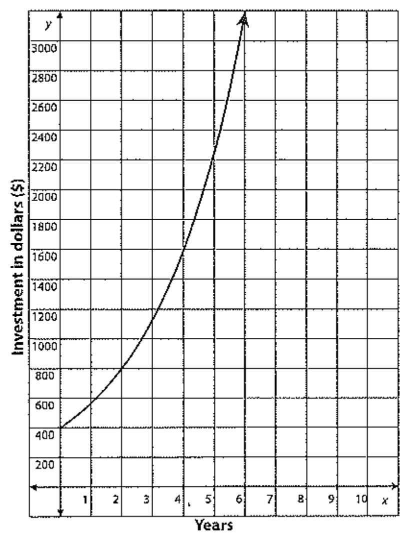
Interpretation:______________________________

____

____

Example 2:

Jasper has invested an amount of money into a savings account. The graph to the right shows the value of his investment over a period of time. What is the rate of change for the interval [1, 3]?



Step 1:

Step 2:

Step 3:

Step 4:

Rate of Change:________.

Step 5: Interpretation:

____

____

(8)

Example 3:

Jasper is curious about how the rate of change differs for the interval [3, 6]. Calculate the rate of change using the graph from Example 2.

Rate of Change:____________.

Interpretation:______________________________

____

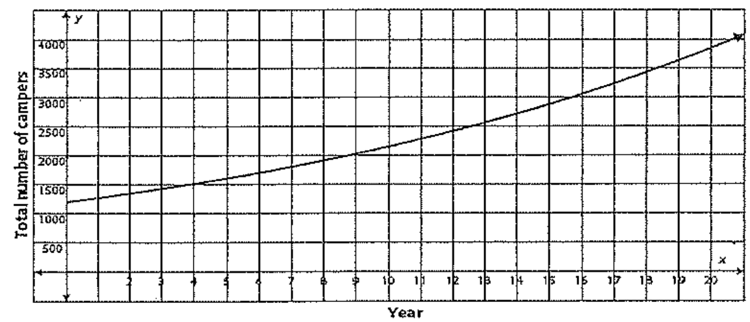
What can you conclude about Jasper's investment value over time?

____

____

You Try 2

Each year, volunteers at a three-day music festival record the number of people who camp on the festival grounds. The graph below shows the number of campers for each of the last 20 years. Calculate the average rate of change over the intervals [3,9] and [9,16].



Rate of Change:____________

Interpretation:______________________________

____

Rate of Change:____________

Interpretation:______________________________

____

(9)

Lesson 3.7 HW - Average Rate of Change

Directions: Find the following exponential functions, find the average rate of change for the indicated intervals and interpret your answer. **Show all work!**

Use the table to answer #1-2. The table below represents the worth each year of an initial investment of $650 that earns 3.4% interest compounded quarterly.

Years (x)	Investment value in dollars ($f(x)$)
0	650
2	862.56
4	1144.64
6	1518.96
8	2015.70

1) What is the average rate of change for this function over the interval [0,6]? __________

Interpretation:

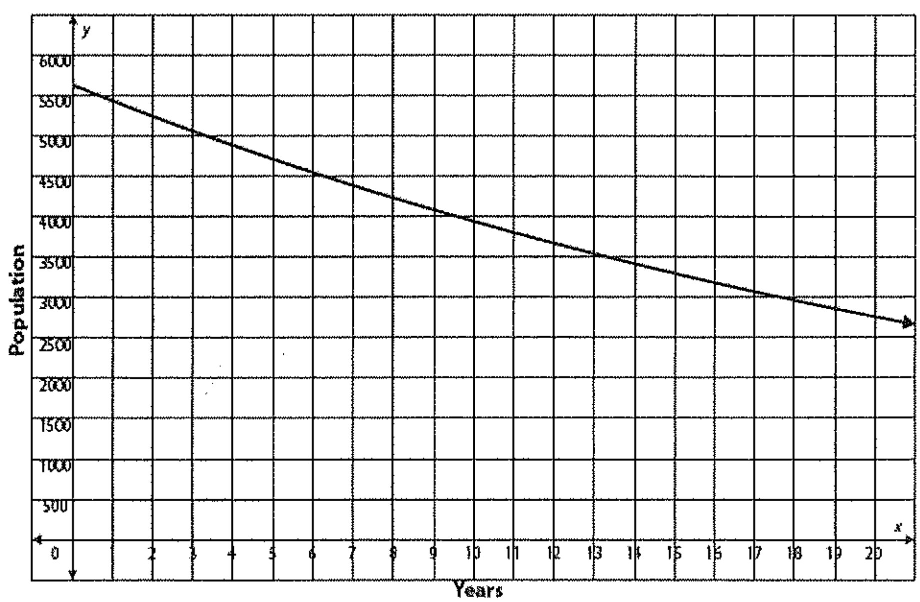
____

2) What is the average rate of change for this function over the interval [4,8]? __________

Interpretation:

____

Use the graph to answer #3-4. The graph below shows the yearly population of a small town.



3) What is the average rate of change for the interval [2,6]?

____

Interpretation:

____

____

4) What is the average rate of change for the interval [9.5,20]?

____

Interpretation:

____

____

10

Lesson 3.8 CW - Building Functions

Introduction

- What is function notation? ____________________
  - Example: $y = 2x + 2$ ______ $= 2x + 2$
- Some functions have characteristics of ____________ ____________ so they are made up of ________ ________ one function.
- In this lesson, you will learn how to create a new function by ________, ________, ________, or ________ ________ ________.

Adding & Subtracting Functions

Example 1: Given two functions, $f(x) = 3x + 3$ and $g(x) = 2x - 7$, add the two functions together.

- $(f + g)(x) = f(x) + g(x)$

  = ____________________

  = ____________________

- $f(x) + g(x) =$ ____________

Example 2: Given two functions, $h(x) = -5x + 2$ and $j(x) = -8x$, subtract the two functions together.

- $(h - j)(x) = h(x) - j(x)$

  = ____________________

  = ____________________

  = ____________________

- $h(x) - j(x) =$ ____________

Example 3: Given two functions, $d(x) = 3^x - 4$ and $a(x) = x + 9$, add the two functions together.

- $(d + a)(x) = d(x) + a(x)$

  = ____________________

  = ____________________

- $d(x) + a(x) =$ ____________

You Try!

Given $f(x) = 6x + 3$ and $g(x) = -2x - 4$:

1. Find $(f + g)(x)$.

2. Find $(f - g)(x)$.

Example 4: Evaluate two functions, $h(x) = 3x + 2$ and $b(x) = 2x - 7$, at $x = 3$.

$h(x) = 3x + 2$

$h(3) =$ ________

= ________

$b(x) = 2x - 7$

$b(3) =$ ________

= ________

- Now let's add the two functions $h(x) = 3x + 2$ *and* $b(x) = 2x - 7$

  $(h + b)(x) = h(x) + b(x)$

  = ____________________

  = ____________________

  $(h + b)(x) =$ ____________________

  Now let's evaluate $(h + b)(3) =$ ____________________

- So by ________ $h(x)$ and $b(x)$ at $x = 3$ and then ________ them together, we get the ________ ________ as adding the ______ ________ together and then evaluating the new function at $x = 3$.

Example 5: If $a(x) = 2x - 3$ and $p(x) = 4x - 11$, what is the result of subtracting the two functions? What is $(a - p)(x) = a(x) - p(x)$?

- $(a - p)(x) =$ ____________

  = ____________

  = ____________

- $(a - p)(x) =$ ____________

x	a(x)	p(x)	(a-p)(x)
-1			
0			
1			
2			

Example 6: If $s(x) = 5^x + 10$ and $t(x) = -2x - 6$, what is the result of subtracting the two functions? What is $(s - t)(x) = s(x) - t(x)$?

- $(s - t)(x) =$ ____________________

  = ____________________

  = ____________________

- $(s - t)(x) =$ ____________________

x	s(x)	t(x)	(s-t)(x)
-1			
0			
1			
2			

- What is $s(2) - t(2)$?
- What is $s(-1) - t(0)$?

You Try!

Given two functions $q(x) = -7x + 3$ and $r(x) = 4x - 1$, find the following:

3. $q(5) + r(6)$

4. $r(-3) - q(2)$

Multiplying Functions

Example 7: If $f(x) = 4x - 1$ and $g(x) = -3x + 6$, what is the result of multiplying the two functions? What is $f(x) * g(x)$?

x	f(x)	g(x)	$f(x) * g(x)$
-2			
-1			
0			
1			

- What is $f(-1) * g(-1)$?
- What is $f(0) * g(-2)$?

Example 8: If $j(x) = 3x - 5$ and $z(x) = -3^x + 2$, what is the result of multiplying the two functions? What is $j(x) * z(x)$?

x	j(x)	z(x)	$j(x) * z(x)$
-2			
-1			
0			
1			

- What is $j(-1) * z(1)$?
- What is $j(0) * z(-2)$?

Dividing Functions

Example 9: If $c(x) = -5x + 2$ and $k(x) = x - 6$, what is the result of dividing the two functions? What is $c(x)/k(x)$?

x	c(x)	k(x)	$c(x)/k(x)$
-2			
-1			
0			
1			

- What is $c(-1)/k(-1)$?
- What is $c(0)/k(-2)$?

You Try!

Given $m(x) = -3x + 2$ and $n(x) = 4x - 1$:

5. Find $m(2) * n(3)$.

6. Find $n(5)/m(1)$.

Lesson 3.8 HW - Building Functions

#1-4 Use $f(x) = 8x - 5$ and $h(x) = x + 4$

1. $f(x) + h(x)$

2. $h(x) - f(x)$

3. $h(3) + f(0)$

4. $f(-2) - h(5)$

#5-6 Use $g(x) = 5^x - 1$ and $e(x) = -4x + 7$

5. $g(x) + e(x)$

6. $g(x) - e(x)$

#7-10 Complete the table below and then use it to answer the questions that follow.

x	$r(x) = -x + 9$	$s(x) = 2x$
-2		
-1		
0		
1		
2		

7. $r(2) * s(0)$

8. $\frac{r(-1)}{s(1)}$

9. $3(r(0)) * s(-2)$

10. $5(s(4)) * s(-1)$

Lesson 3.9 - Word Problems with Exponential Functions

Exponential Growth

Formula:

- The percent of __________ is _______.
- Remember if b > 1, then you will have _________.

Steps to solving word problems with exponential functions
1. ________ the __________ you're using. 2. __________ the needed quantities into your formula. 3. _________ the formula. 4. __________ your answer.

Example 1

A population of 40 pheasants is released in a wildlife preserve. The population doubles each year for 3 years. What is the population after 4 years?

1. Formula:

2. Substitute:

3. Evaluate:

4. Interpretation:

You Try 1

A population of 50 pheasants is released in a wildlife preserve. The population triples each year for 3 years. What is the population after 3 years?

Exponential Growth (Money)

Formula:

- The percent of increase is 100r.
- Remember if $b > 1$, then you will have growth.

Example 2

A principal of $600 is deposited in an account that pays 3.5% interest compounded yearly. Find the account balance after 4 years.

1. Formula:
2. Substitute:
3. Evaluate:

4. Interpretation:

____

____

____

You Try 2

A principal of $450 is deposited in an account that pays 2.5% interest compounded yearly. Find the account balance after 2 years.

You Try 3

A principal of $800 is deposited in an account that pays 3% interest compounded yearly. Find the account balance after 5 years.

Exponential Decay

Formula:

- The percent of ___________ is ________.
- Remember if $0 < b < 1$, then you will have __________.

Example 3

You bought a used truck for $15,000. The value of the truck will decrease each year because of depreciation. The truck depreciates at the rate of 8% per year. Estimate the value of your truck in 5 years.

1. Formula:
2. Substitute:
3. Evaluate:

4. Interpretation:

____

____

____

You Try 4

Use the exponential decay model in example 3 to estimate the value of your truck in 7 years.

You Try 5

Rework example 3 if the truck depreciates at the rate of 10% per year.

Annual Percent of Increase or Decrease

- The annual percent of increase or decrease comes from the Growth and Decay ________ of the exponential formulas.
- Identify the growth and decay factors in the formula
  $y = C(1-r)^t$ $y = C(1+r)^t$
  Which factor is growth? ________
  Which factor is decay? ________

Steps for finding the Annual Percent of Increase or Decrease

- Step 1: Identify if the function is a ________ or a ________.
- Step 2: Write the factor from the corresponding exponential formula and set it equal to the base. Growth: ____________ Decay: ____________
- Step 3: Solve the formula for ____.
- Step 4: Find the percent of increase or decrease. Use your answer from step 3 and plug it into ______.

Example 4

Find the annual percent of increase or decrease that $f(x) = 2(1.25)^x$ models.

1. Identify if it's a growth or decay.

____

2. Write the factor and set it equal to the base.

3. Solve for r

4. Find the percent of increase or decrease using 100r.

Example 5

Find the annual percent of increase or decrease that $f(x) = 3(0.80)^x$ models.

1. Identify if it's a growth or decay.

____

2. Write the factor and set it equal to the base.

3. Solve for r

4. Find the percent of increase or decrease using 100r.

You Try 6-8 Directions: Find the annual percent of increase or decrease that the given exponential functions model

You Try 6 $f(x) = 3(.54)^x$

You Try 7. $f(x) = 2\,(1.35)^x$

You Try 8. $f(x) = 4\,(.67)^x$