

Name: _____ Period _____

Unit 3C:

Sequences

Unit 3: Linear and Exponential Functions

Resources and Information at:

www.tinyurl.com/9dayson



Unit 3C: Sequences

Unit Essential Question: How do we recognize and create tables, graphs, and formulas to describe arithmetic and geometric sequences?

<u>Concept:</u>	<u>Concept:</u>	<u>Concept:</u>
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<u>Lesson Essential Questions:</u>	<u>Lesson Essential Questions:</u>	<u>Lesson Essential Questions:</u>
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<u>Vocabulary:</u>	<u>Vocabulary:</u>	<u>Vocabulary:</u>
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Additional Information/Resources:

www.usatestprep.com school: newmanchesterga
code: newton58

Lesson 3.10 - Intro to Sequences

_____ - A list of numbers or objects in a special order.

- Each value in the sequence is called a _____.
- A sequence can be _____ (with no end) or _____ (ends).
- A sequence can be written as a _____.
- We use A_n, U_n , etc. to represent sequences.
 - n acts like the _____ or x-value (x)
 - A_n acts like the _____ or y-value ($f(x)$)
 - A_3 is another way to say the _____ term
- The _____ of the function that generates a sequence is all _____ numbers.

We will work with two types of sequences:

- _____ - a sequence that increases or decreases by a constant rate
(you _____ or _____)
- _____ - a sequence that increases or decreases by a factor
(you _____)

Example 1: $A_n = 1, 3, 5, 7, \dots$

1. Pattern: _____
 Type: _____
 2. Infinite or Finite? _____
 3. Next term: _____
 4. Second term: _____ 5. A_5 : _____

Example 2: $A_n = 2, 4, 8, 16, \dots$

1. Pattern: _____
 Type: _____
 2. Infinite or Finite? _____
 3. Next term: _____
 4. Second term: _____ 5. A_5 : _____

2

Example 3: $A_n = 1, 1, 2, 3, 5, 8, \dots$

1. Pattern: _____
 Type: _____
 2. Infinite or Finite? _____
 3. Next term: _____
 4. Second term: _____ 5. A_5 : _____

Example 4:

n	1	2	3	4	5
A_n	12	8	4	0	2

1. Pattern: _____
 Type: _____
 2. Next term: _____
 3. Second term: _____ 4. A_5 : _____

You Try 1: $A_n = 0, 5, 10, 15, \dots, 25$.

1. Pattern: _____
 Type: _____
 2. Infinite or Finite? _____
 3. Next term: _____
 4. Second term: _____ 5. A_5 : _____

Example 5:

n	1	2	3	4	5
A_n	17	15	10	8	5

1. Pattern: _____
 Type: _____
 2. Next term: _____
 3. Second term: _____ 4. A_5 : _____

You Try 2:

n	1	2	3	4	5
A_n	27	9	3	1	2

1. Pattern: _____
 Type: _____
 2. Next term: _____
 3. Second term: _____ 4. A_5 : _____

3

Lesson 3.10 HW - Intro to Sequences

#1-4 a. Describe the pattern.

b. Identify if the sequence is geometric, arithmetic, or neither.

1. $A_n = 35, 32, 29, 26, \dots$

a. _____

b. _____

2. $A_n = 1, 4, 16, 64, \dots$

a. _____

b. _____

3. $A_n = 6, 36, 6, 36, \dots$

a. _____

b. _____

4. $A_n = -7, -9, -11, -13, \dots$

a. _____

b. _____

#5-8 a. Find the next term b. Find A_2

5. $A_n = 16, 4, 1, \frac{1}{4}, \dots$

a. _____ b. _____

6. $A_n = -30, -40, -50, -60, \dots$

a. _____ b. _____

7.

n	1	2	3	4	5
A_n	24	19	14	9	?

a. _____ b. _____

8.

n	1	2	3	4	5
A_n	-3	-15	-75	-375	?

a. _____ b. _____

#9-10 Use the Fibonacci Sequence 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, ...

9. What is the pattern in the Fibonacci Sequence?

10. What would the next term in the sequence be?

Lesson 3.11 - Arithmetic Sequences

An arithmetic sequence is a list of terms separated by a _____, d , which is the number added to each consecutive term in an arithmetic sequence.

An arithmetic sequence is a linear function with a _____ of whole numbers.

_____ can be represented by formulas, either explicit or recursive, and those formulas can be used to find a certain term of the sequence or the number of a certain value in the sequence.

- A _____ is a formula used to find the next term of a sequence when the previous term is known.
- An _____ is a formula used to find the n th term of a sequence.

Explicit Formula: "Finds a specific term"

Recursive Formula: "Uses previous terms to find the next term"

Example 1 Consider the sequence 3, 6, 9, 12, 15, 18, ...

Find the following terms: $a_2 =$ _____; 5th Term = _____; $a_6 =$ _____

You Try 1-4

Consider the sequence -7, -2, 3, 8, ... Find the following terms:

1. $a_1 =$ _____
2. Third Term = _____
3. Fifth Term = _____
4. $a_7 =$ _____

Example 2

Create the recursive formula that defines the sequence:

An arithmetic sequence is defined by 8, 1, -6, -13, ...

1. Find the common difference, d .

- The sequence is _____ so d will be _____
- $d =$ _____

2. Use the recursive formula: $A_n = A_{n-1} \pm d$

Recursive Formula: _____

Example 3

Create the recursive formula that defines the sequence:

An arithmetic sequence is defined by: 10, 6, 2, -2, ...

$d =$ _____; Recursive Formula: _____

You Try 5

Use the following sequence to create a recursive formula. $A_n = A_{n-1} \pm d$

18, 10, 2, -6, ...

$d =$ _____; Recursive Formula: _____

Example 4

An arithmetic sequence is defined recursively by $a_n = a_{n-1} + 5$, with $a_1 = 29$. Find the first 5 terms of the sequence.

- | | |
|---------------|---------------|
| $a_1 =$ _____ | $a_3 =$ _____ |
| $a_2 =$ _____ | $a_4 =$ _____ |
| $a_5 =$ _____ | $a_5 =$ _____ |

The first five terms of the sequence are _____, _____, _____, _____, and _____

(5)

(6)

Example 5

An arithmetic sequence is defined recursively by $a_n = a_{n-1} - 8$, with $a_1 = 68$. Find the first 5 terms of the sequence.

The first five terms of the sequence are: _____, _____, _____, _____, and _____.

You Try 6

An arithmetic sequence is defined recursively by $a_n = a_{n-1} - 4$, with $a_1 = 12$.

Find the first 5 terms of the sequence.

_____, _____, _____, _____, and _____.

Example 6

Write an explicit formula to represent the sequence from example 4, and find the 15th term.

The first five terms of the sequence are 29, 34, 39, 44, and 49.

1. The first term is $a_1 =$ _____ and the common difference is $d =$ _____.

$$a_n = d(n - 1) + a_1$$

Explicit Formula: _____

2. Simplify.

$$a_n = 5(n - 1) + 29$$

Distribute the 5

$$a_n = 5n + 24$$

3. Substitute 15 in for n to find the 15th term in the sequence:

The 15th term in the sequence is _____.

7

Example 7: Write an explicit formula to represent the sequence from example 2, and find the 12th term. An arithmetic sequence is defined by 8, 1, -6, -13, ...

$$a_n = d(n - 1) + a_1$$

1. The first term is $a_1 =$ _____ and the common difference is $d =$ _____.

2. Simplify.

Explicit Formula

Distribute the -7

Combine Like Terms

3. Substitute 12 in for n to find the 12th term in the sequence.

The 12th term in the sequence is _____.

You Try 7

Use the following sequence to create an explicit formula. $a_n = d(n - 1) + a_1$. Then find a_{20} .

18, 10, 2, -6, ...

$$a_1 = \text{_____} \quad d = \text{_____}$$

Explicit Formula _____

Simplified Explicit Formula _____

The 20th term in the sequence is _____.

8

Lesson 3.11 HW - Arithmetic Sequences

#1-4 Given the sequence: -5, 0, 5, 10, ... Identify the following terms:

1. 2nd Term

2. a_3

3. a_6

4. Seventh Term

#5-6 Given the following sequences:

a. Create a recursive formula: $A_n = A_{n-1} \pm d$

b. Find the next 5 terms.

5. $A_1 = 35$ $d = -3$

6. -4, -16, -28, -40, ...

#7-8 Given the following sequences:

a. Create the explicit formula: $A_n = d(n - 1) + a_1$

b. Find the 33rd term.

7. -1, 6, 13, 20, ...

8. -7, -9, -11, -13, ...

#9-10 Given the following sequences:

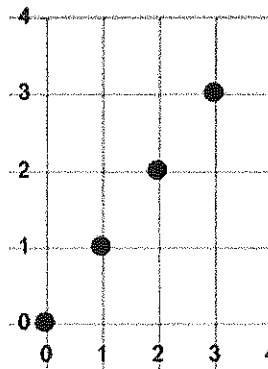
a. Create the explicit formula: $A_n = d(n - 1) + a_1$

b. Find the 21st term.

9.

n	A_n
1	12
2	15
3	18
4	21

10.



Lesson 3.12 - Geometric Sequences

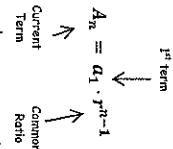
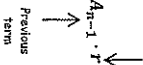
Introduction

- A geometric sequence is a list of terms separated by a _____, r , which is the number multiplied by each consecutive term in a geometric sequence.
- A geometric sequence is an _____ with a domain of _____ in which the ratio between any two consecutive terms is equal.
- Just like arithmetic sequences, _____ can be represented by formulas, either explicit or recursive, and those formulas can be used to find a certain term of the sequence or the number of a certain value in the sequence.

Recall

- A _____ is a formula used to find the next term of a sequence when the previous term is known.
- An _____ is a formula used to find the n th term of a sequence.

Geometric Sequences Formulas and Steps

- Explicit Formula: $A_n = a_1 \cdot r^{n-1}$: This formula finds a specific term

- Recursive Formula: $A_n = A_{n-1} \cdot r$; This formula uses previous terms to find the next terms


Steps to create formulas and solve for geometric sequences

- Find the common ratio by _____ the _____ term by the _____ term.
- _____ which formula to use. (explicit or recursive)
- _____ your values to create your formula.
- Find the _____ term by evaluating if asked to do so.

10

Example 1

Create a recursive formula that defines the sequence. A geometric sequence is defined by 2, 8, 32, 128,

Step 1: Find the common ratio	Step 3: Substitute what you have
Step 2: Explicit or Recursive Formula?	

Example 2

Create the recursive formula that defines the sequence. A geometric sequence is defined by

45, -15, 5, $-\frac{5}{3}$, ...

Step 1: Find the common ratio	Step 3: Substitute what you have
Step 2: Explicit or Recursive Formula?	

You Try 1

Use the following sequence to create a recursive formula. $A_n = A_{n-1} \cdot r$
 10, -30, 90, -270, ...

Step 1: Find the common ratio	Step 3: Substitute what you have
Step 2: Explicit or Recursive Formula?	

11

Example 3

A geometric sequence is defined recursively by $a_n = a_{n-1} \cdot -3$, with $a_1 = 6$. Find the first 5 terms of the sequence.

You Try 2

An arithmetic sequence is defined recursively by $a_n = a_{n-1} + 6$, with $a_1 = 0.2$. Find the first 5 terms of the sequence

Example 5

Write an explicit formula to represent the sequence from example 1, and find the 10th term. The first five terms of the sequence are: 2, 8, 32, 128, and 512.

Step 1: Find the common ratio & a_1	Step 3: Substitute what you have
Step 2: Explicit or Recursive Formula?	Step 4: Evaluate for specific term

Example 6

Write an explicit formula to represent the sequence from example 3, and find the 15th term. The first five terms of the sequence are: 6, -18, 54, -162, and 486

Step 1: Find the common ratio & a_1

Step 3: Substitute what you have

Step 2: Explicit or Recursive Formula?

Step 4: Evaluate for specific term

Step 1: Find the common ratio & a_1	Step 3: Substitute what you have
Step 2: Explicit or Recursive Formula?	Step 4: Evaluate for specific term

You Try 3

Use the following sequence to create an explicit formula. $a_n = a_1 \cdot r^{n-1}$ Then find a_{14} . -4, 8, -16, 32, ...

Step 1: Find the common ratio & a_1

Step 3: Substitute what you have

Step 2: Explicit or Recursive Formula?

Step 4: Evaluate for specific term

Step 1: Find the common ratio & a_1	Step 3: Substitute what you have
Step 2: Explicit or Recursive Formula?	Step 4: Evaluate for specific term

12

13

Lesson 3.12 HW - Geometric Sequences

#1-2 Create the recursive formula: $A_n = A_{n-1} \cdot r$ that defines the given sequence.

1. 7, -21, 63, -189, ...

2. 150, 30, 6, 1.2, ...

#3-4 Use the recursive formula to write the first 5 terms of the sequence.

3. $A_n = A_{n-1} \cdot 8$; $a_1 = 5$

4. $A_n = A_{n-1} \cdot -\frac{1}{2}$; $a_1 = 16$

#5-6 Given the following sequences:

a. Create the explicit formula: $A_n = a_1 \cdot r^{n-1}$

b. Find a_{10}

5. 9, 18, 36, 72, ...

6. $a_1 = 0.2$; $r = -5$

7-8 Use the graph or chart to:

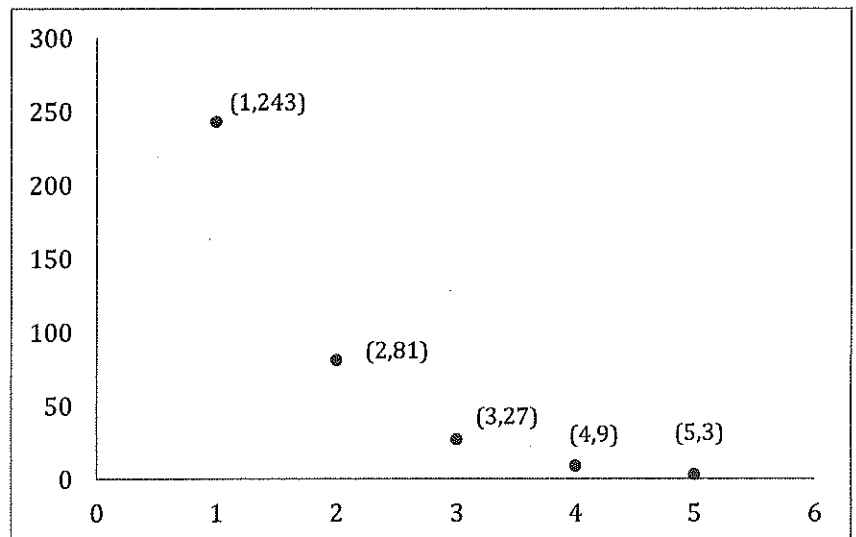
a. Create the explicit formula: $A_n = a_1 \cdot r^{n-1}$

b. Find a_{13}

7.

n	a_n
1	$\frac{1}{4}$
2	$\frac{1}{2}$
3	1
4	2
5	4

8.



Lesson 3.13 - Applications of Arithmetic Sequences

- In a word problem, look for a _____ being used between each term.

Steps to Finding a Term for an Arithmetic Sequence

1. Create a _____ of the word problem.
2. _____ the sequence in _____ to identify the common difference, _____, and the first term, _____.
3. Determine which _____ would best fit the situation (_____ or _____)
4. _____ d and a_1 in to the formula from step 3.
5. _____ the formula for the given term.
6. _____ the result.

Example 1: You visit the Grand Canyon and drop a penny off the edge of a cliff. The distance the penny will fall is 16 feet the first second, 48 feet the next second, 80 feet the third second, and so on in an arithmetic sequence. What is the total distance the object will fall in 6 seconds?

Picture	Sequence
Formula	Interpretation

(15)

Example 2: Tom just bought a new cactus plant for his office. The cactus is currently 3 inches tall and will grow 2 inches every month. How tall will the cactus be after 14 months?

Picture	Sequence
Formula	Interpretation

Example 3: Kayla starts with \$25 in her allowance account. Each week that she does her chores, she receives \$10 from her parents. Assuming she doesn't spend any money, how much money will Kayla have saved after 1 year?

Picture	Sequence
Formula	Interpretation

(16)

You Try!

A theater has 26 seats in row 1, 29 seats in row 2, and 32 seats in row 3 and so on. If this pattern continues, how many seats are in row 42?

Picture	Sequence
Formula	Interpretation

Lesson 3.12 HW – Applications of Arithmetic Sequences

Directions: Use the following problems to complete the graphic organizers.

1. A company began doing business four years ago. Its profits for the last 4 years have been \$10 million, \$15 million, \$20 million and \$25 million. If the pattern continues, what is the expected profit in 26 years?

Picture	Sequence
Formula	Interpretation

17

2. A production line is improving its efficiency through training and experience. If the number of items produced in the first four days of a month are 4, 10, 16, and 22, respectively. Project the number of items produced by the end of a 30 day month if the pattern continues.

Picture	Sequence
Formula	Interpretation

3. You visit Niagara Falls and drop a penny off the edge of the waterfall. The distance the penny will fall is 7 feet the first second, 15 feet the next second, 23 feet the third second, and so on in an arithmetic sequence. What is the total distance the object will fall in 12 seconds?

Picture	Sequence
Formula	Interpretation

18

Lesson 3.14 - Applications of Geometric Sequences

- In a word problem, look for a _____ being used between each term.

Steps to Finding a Term for a Geometric Sequence

- Create a _____ of the word problem.
- Write out the sequence in order to _____ the common _____, r and the _____ term, a_1 .
- Determine which _____ would best fit the situation (Recursive or Explicit)
- _____ r and a_1 into the formula from step 3.
- _____ the formula for the given term.
- _____ the result.

Example 1: On a Saturday, you sit in your yard and decide to count the number of dandelions that you see. You count a total of 39 dandelions. On Sunday, you count 78 dandelions and on Monday you count 156. If this pattern continues, how many dandelions will there be in your yard on Friday?

Picture	Sequence
Formula	Interpretation

19

Example 2: Suppose you drop a ping pong ball from a height of 20 feet. After the ball hits the ground, it rebounds to 85% of its previous height. How high will the ball rebound after its third bounce? Round your answer to the nearest tenth.

Picture	Sequence
Formula	Interpretation

Example 3: You and your family go on vacation and stay at a hotel. You go down to the pool area to sock in the hot tub. To your disappointment, the water is freezing cold. You complain to the manager and she tells you that she will increase the temperature by 10% each hour. If the current temperature of the hot tub is 65° F, what will be the temperature of the hot tub after 4 hours, to the nearest tenth of a degree?

Picture	Sequence
Formula	Interpretation

20

You Try!

You open a savings account with \$350 that you have saved. The bank offers 2.8% simple interest each year. How much money will you have in your account after 10 years?

Picture	Sequence
Formula	Interpretation